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# Terror Queues and the Duration of Terror Plots

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New Haven, Connecticut

IIASA, Laxenburg, December 18, 2013

# Motivation

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- ◆ “Intelligence is the heart and soul of operational counterterrorism” (Amos Guiora (2008), *Fundamentals of Counterterrorism*)
- ◆ Terror queues model is part of a research program devoted to “intelligence operations research”
- ◆ Focus on undercover agents and/or informants; goal is to understand the interaction of human intelligence with the detection and interdiction of terror plots

# Motivation

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- ◆ Reduce the rate of successful terror attacks
- ◆ Working at **operations/tactical** level as opposed to foreign policy, economic assistance etc.

## Intelligence to Counter Terrorism: Issues for Congress

Observers point to several major challenges that the Intelligence Community will likely encounter in supporting the counter terrorist effort.

- **First is a renewed emphasis on human agents.** Signals intelligence and imagery satellites have their uses in the counterterrorism mission, but intelligence to counter terrorism depends more on human intelligence (humint) such as spies and informers.

**Source:** Best RA, Congressional Research Service Report RL31292, 2003,  
<http://www.au.af.mil/au/awc/awcgate/crs/rl31292.pdf>

# Motivation: Intelligence Counterterrorism Link

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- ◆ Consider terror attacks targeting Israeli civilians during the Second Intifada
- ◆ Strong statistical evidence that HUMINT-driven IDF “preventive events” (arrests) are associated with reduction in suicide bombing attempts
  - Kaplan *et al* 2005. *Studies in Conflict and Terrorism* 28:225-235
  - Kaplan *et al* 2006. *Interfaces* 36(6):553-561
- ◆ How do HUMINT and counterterror operations enable such results?



Mosab  
Hassan  
Yousef.  
(Yossi  
Sasson)

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## Haaretz exclusive: Hamas founder's son worked for Shin Bet for years

By [Avi Issacharoff](#)

Tags: [Israel News](#), [Shin Bet](#), [Hamas](#)

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The son of a leading Hamas figure, who famously converted to Christianity, served for over a decade as the Shin Bet security service's most valuable source in the militant organization's leadership, Haaretz has learned.

Mosab Hassan Yousef is the son of Sheikh Hassan Yousef, a Hamas founder and one of its leaders in the West Bank. The intelligence he supplied Israel led to the exposure of a number of terrorist cells, and to the prevention of dozens of suicide bombings and assassination attempts on Israeli figures.

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## N.Y. / Region

# Man Is Charged With Plotting to Bomb Federal Reserve Bank in Manhattan

By MOSI SECRET and WILLIAM K. RASHBAUM


Published: October 17, 2012


Federal prosecutors in Brooklyn charged a 21-year-old Bangladeshi man with conspiring to blow up the Federal Reserve Bank of New York, saying he tried to remotely detonate what he believed was a 1,000-pound bomb in a van he parked outside the building in Lower Manhattan on Wednesday.





But the entire plot played out under the surveillance of the [Federal Bureau of Investigation](#) and the New York Police Department as part of an elaborate sting operation, according to [court papers](#).


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THE  
SESSIONS  
COMING SOON

OP-ED CONTRIBUTOR

## The Terrorist Next Door

By MICHAEL A. SHEEHAN

Published: May 3, 2010

SIGN IN TO

So law enforcement has to focus on preventing sophisticated terrorist organizations from establishing a presence within the United States. The good news is that we know how to do this. The bad news is we aren't doing it enough. No other American city even attempts to do what New York has accomplished. The New York Police Department's intelligence and counterterrorism units, working both with the F.B.I. and independently, manage a network of informant and undercover operatives around the area. It was no accident that last year when a Denver man who was planning to bomb the New York subway system arrived in the city, the F.B.I. was aware of his travels, and a radical cleric he met with was already a police informant.



# Motivation

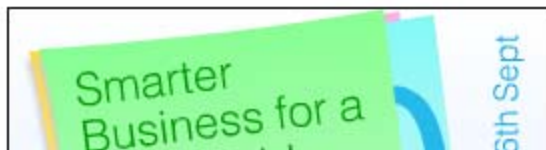


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## Europe terror alert fuelled by German militants

By Stephen Brown BERLIN | Tue Oct 5, 2010 2:22pm BST

With papers picturing Berlin landmarks named as potential targets, including the Brandenburg Gate and the Fernsehturm (TV tower) that dominates the skyline, the head of the main German police union warned: "We should expect attacks."

"The number of dangerous Islamists (in Germany) lies at more than 100," Konrad Freiberg, chairman of the union, told the Passauer Neue Presse newspaper, adding that about 40 had explosives training. "This is very dangerous for us."

European and American counter-terrorism officials have also said that concerns about a group of about 100 German Islamists who had travelled between Germany and tribal border areas of Pakistan contributed to the latest security alert in Europe.

# Motivation

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Last Updated: Wednesday, 2 May 2007, 13:53 GMT 14:53 UK

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## MI5 watch 2,000 terror suspects

By Frank Gardner  
BBC security correspondent

The number of terror suspects being monitored by MI5 in the UK has grown by a quarter in the past six months, the BBC has learned.

The security service and police are monitoring about 2,000 individuals who they say are actively involved in supporting al-Qaeda.

Some are thought to have direct links with al-Qaeda in Pakistan.

The fertiliser bomb plot case has highlighted the links to terrorist training camps in Pakistan.

More than 400,000 Britons each year go to Pakistan on innocent family visits.

But Pakistani intelligence agents cannot follow everyone.



Links between the fertiliser plotters and the 7 July bombers have brought MI5's work into sharp focus

“ I think this is the strongest connection that we are confronted with at the

### FERTILISER BOMB TRIAL

#### LATEST NEWS

- ▶ Terror case 'naysayers' attacked
- ▶ 'Time wasting' in bomb plot trial
- ▶ UK plotters 'wasted their lives'
- ▶ Blair rejects 7/7 inquiry calls
- ▶ Five get life over UK bomb plot

#### PLOT EXPLAINED



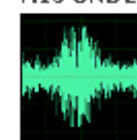
#### How the plot formed - and failed

Who plotters were, how they were linked, what they planned

#### FEATURES & BACKGROUND

- ▶ Q&A: UK terrorism trial
- ▶ In pictures: Men in Pakistan
- ▶ Timeline: Operation Crevice
- ▶ Bomb plot's roots in Pakistan
- ▶ How bad could bomb have been?

#### MI5 UNDER SCRUTINY



#### Revealed: Bomber transcript

Bugged conversation with a suicide bomber.

- ▶ Panorama special investigation



**ALQaeda**  
COMING SOON AGAIN IN  
New York

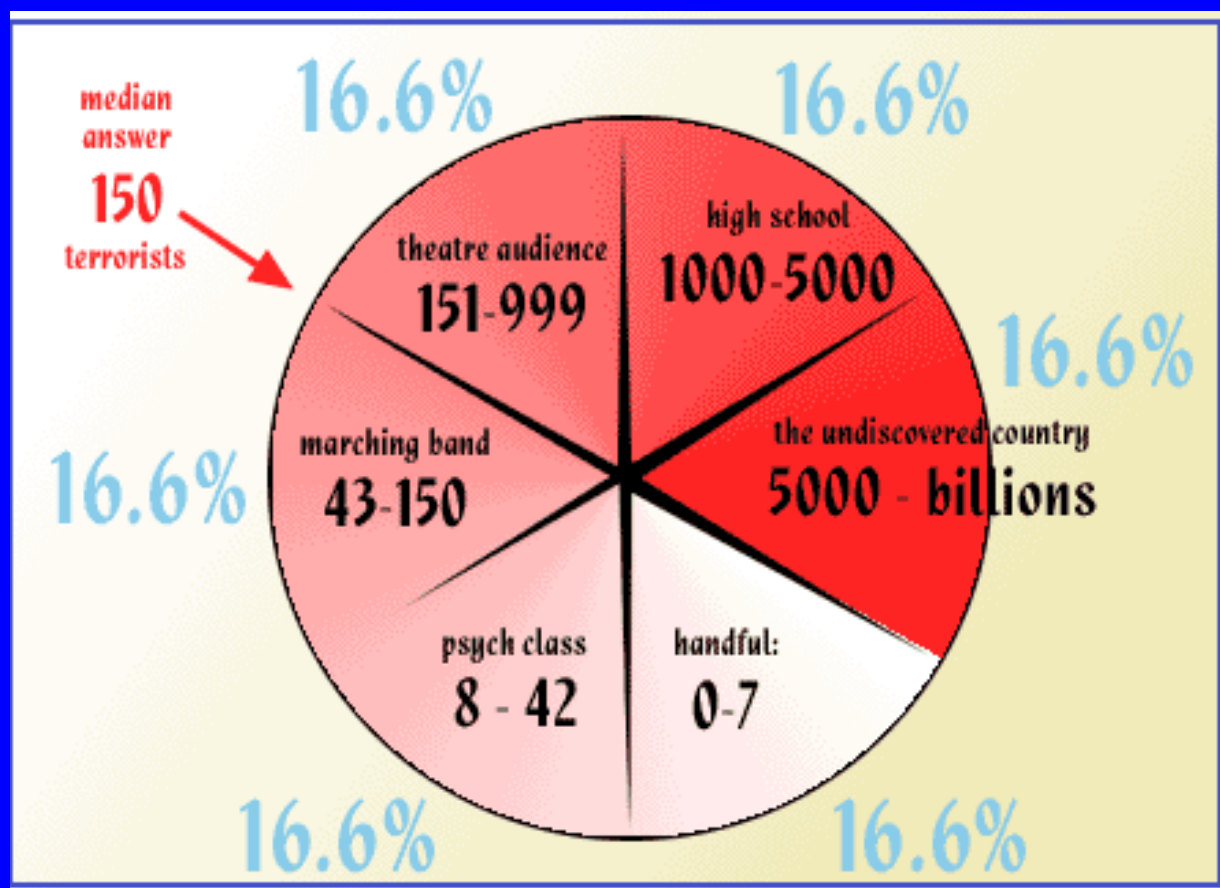
# Motivation

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- ◆ More generally, how might one estimate the extant number of undetected terror plots?

# How Many Terror Plots Are There?

- ◆ Three. And two of them are ex-girlfriends.



[http://www.cockeyed.com/citizen/terror/terror\\_results.html](http://www.cockeyed.com/citizen/terror/terror_results.html)

# In The US...

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- ◆ According to the FBI:
- ◆ <400K people on consolidated terrorism watch list
  - <16K people on selectee (can be singled out for extra inspections) and no fly lists
    - » <2.5K people on no fly lists
- ◆ False positives on no fly lists well known
  - Ted Kennedy, John Lewis, Nelson Mandela, Yusuf Islam (Cat Stevens), ...
- ◆ 600 names removed from consolidated list daily

# Research Contribution

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- ◆ Present analysis models the interaction between terror threats, HUMINT and counterterror operations using queueing theory
- ◆ Model shows how changing the terrorists' operating environment improves the chance that HUMINT detects terror threats
- ◆ Model also enables estimation of the number of undetected terror plots

# Talk Overview

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- ◆ Review terror queue model
- ◆ Use model for staffing problems
- ◆ Duration of *Jihadi* plots in the US
- ◆ Terror queues with non-exponential durations
- ◆ Terror queues with proportional hazards
- ◆ Staffing models with proportional hazards

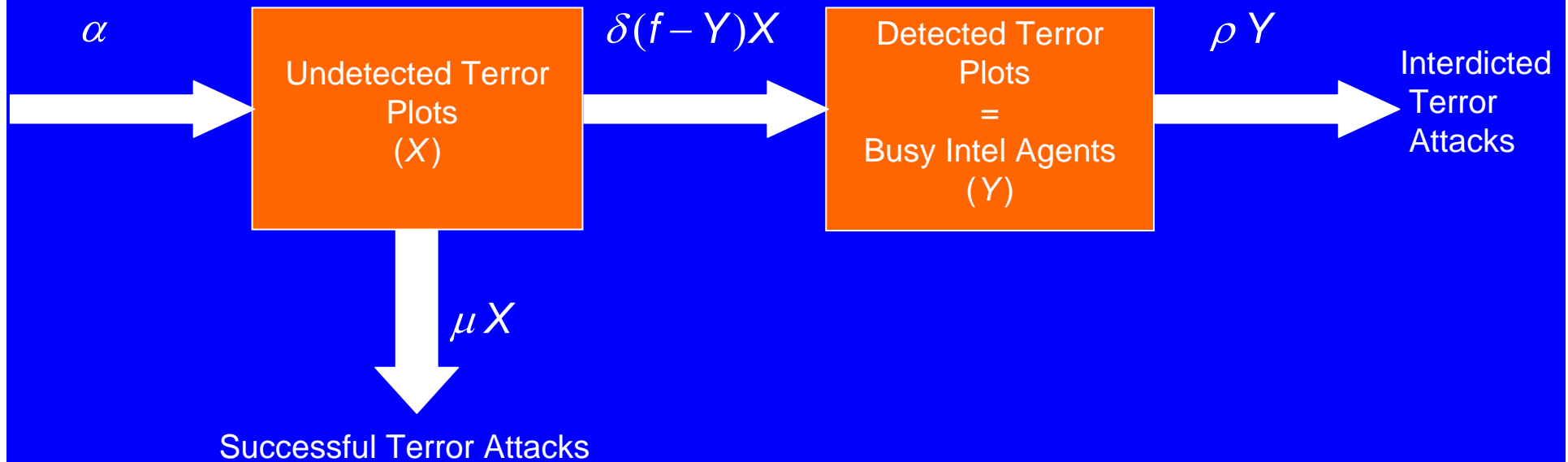


# Terror Queues

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- ◆ Consider terror plots as “customers”
- ◆ Customers arrive (new plots are hatched) in accord with Poisson process
- ◆ “Servers” are undercover agents or informants
- ◆ “Service” commences when a plot is detected by an “available” agent (servers have to find customers), and concludes when the plot is interdicted (agents occupied with specific plots are “busy”)
- ◆ Successful terror plots are equivalent to customers who abandon the queue (drop out) before receiving service
- ◆ *Idle servers and waiting customers co-exist!*
- ◆ *Servers want to provide good service, but customers don't want to be served!*

# Terror Queue Model



## Parameters

$\alpha$  = terror plot arrival rate

$\mu$  = unobstructed terror plot completion rate

$\delta$  = terror plot detection rate

$\rho$  = detected terror plot interdiction rate

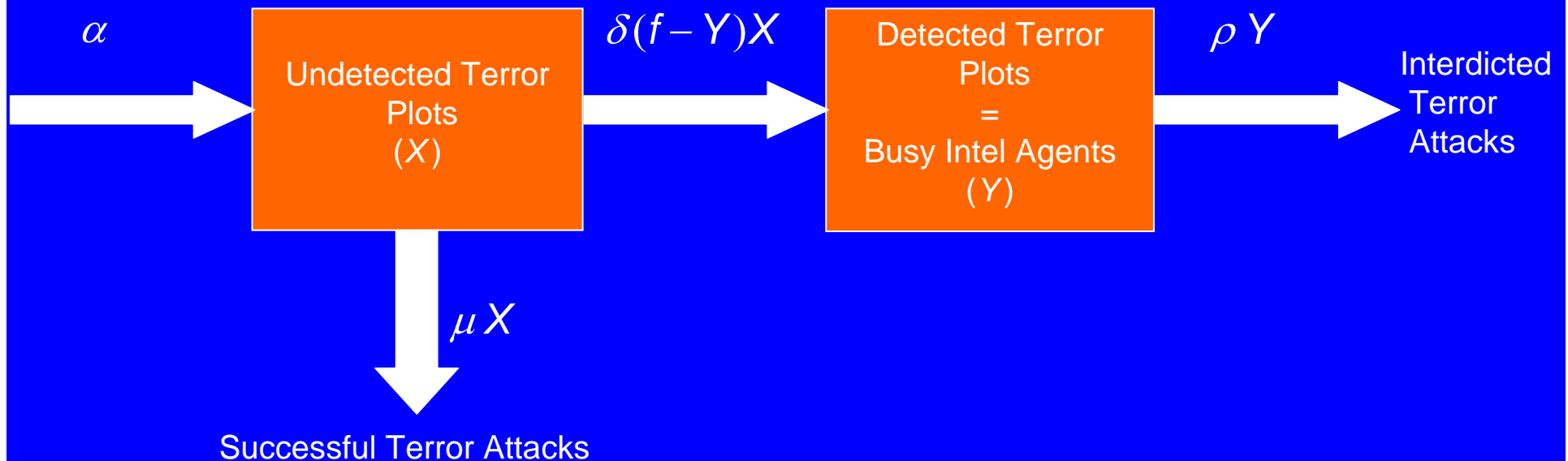
$f$  = total number of intel agents

## State Variables

$X$  = number of undetected terror plots

$Y$  = number of detected terror plots/busy intel agents

# Reducing the Rate of Successful Terror Attacks



◆ Successful terror attack rate =  $\mu E(X)$

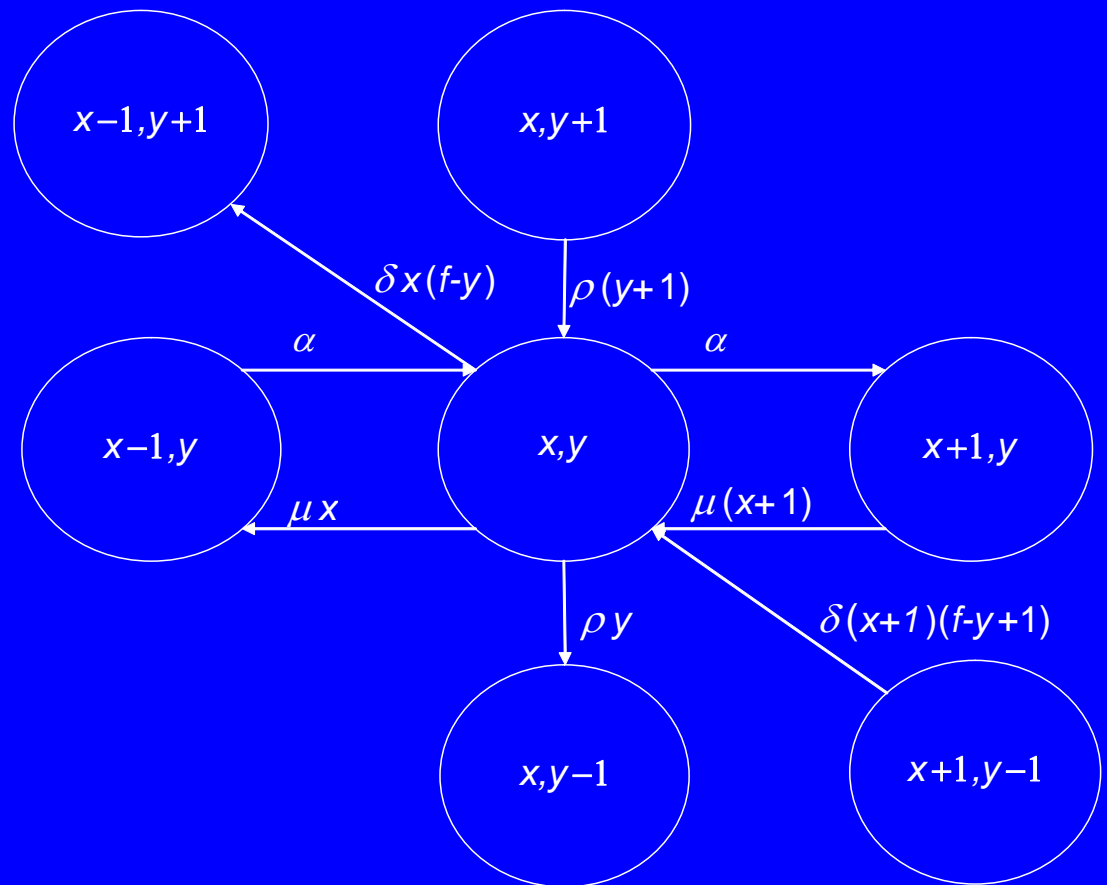
- Delay terrorists (reduce  $\mu$ )
- Improve detection (increase  $\delta$ )
- Speed interdiction (increase  $\rho$ )
- Have lots of agents (increase  $f$ )

Goal: determine the joint probability distribution of undetected ( $X$ ) and detected ( $Y$ ) terror threats:

$$p_{xy} = \Pr\{X=x, Y=y\}$$

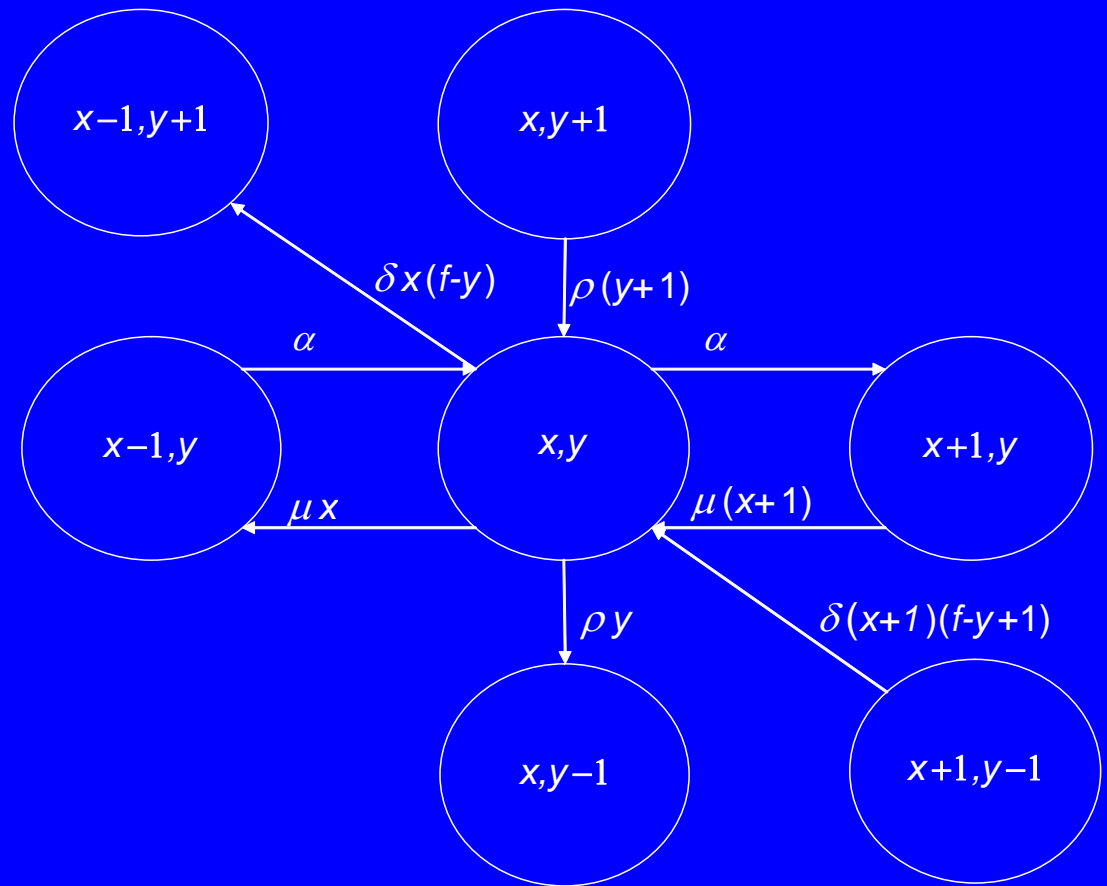
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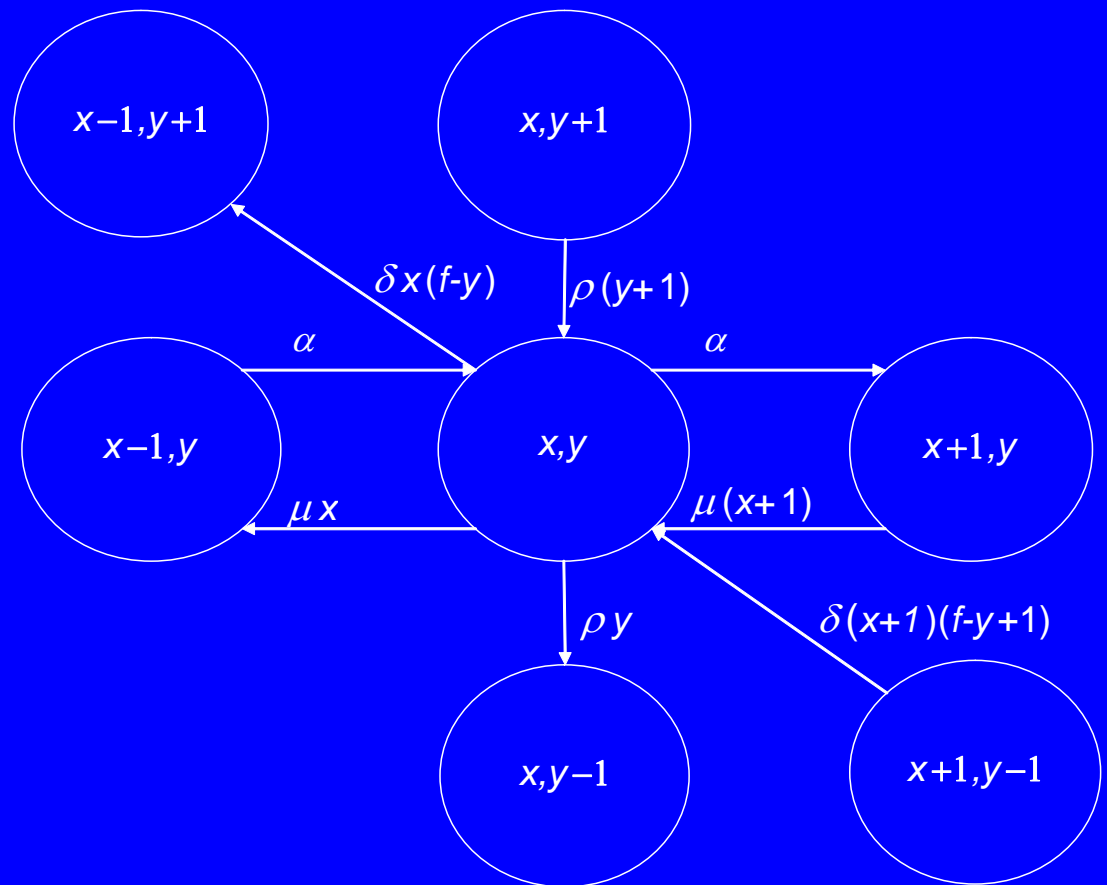
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Generic balance equation:

Goal: determine the joint probability distribution of undetected ( $X$ ) and detected ( $Y$ ) terror threats:

$$p_{xy} = \Pr\{X=x, Y=y\}$$

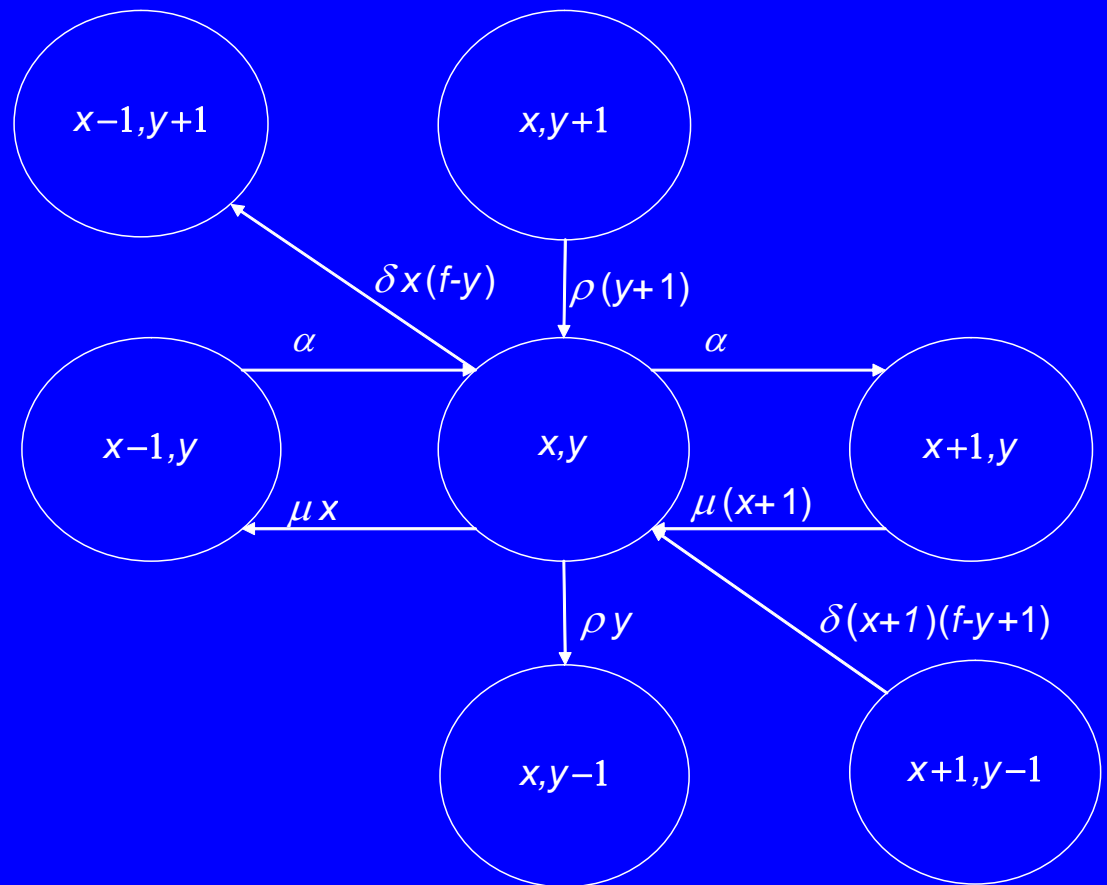


Generic balance equation:

$$\begin{aligned} & (\alpha + \mu x + \rho y + \delta x(f-y))p_{xy} \\ = & \alpha p_{x-1,y} + \mu(x+1)p_{x+1,y} + \rho(y+1)p_{x,y+1} + \delta(x+1)(f-y+1)p_{x+1,y-1} \end{aligned}$$

Goal: determine the joint probability distribution of undetected ( $X$ ) and detected ( $Y$ ) terror threats:

$$p_{xy} = \Pr\{X=x, Y=y\}$$



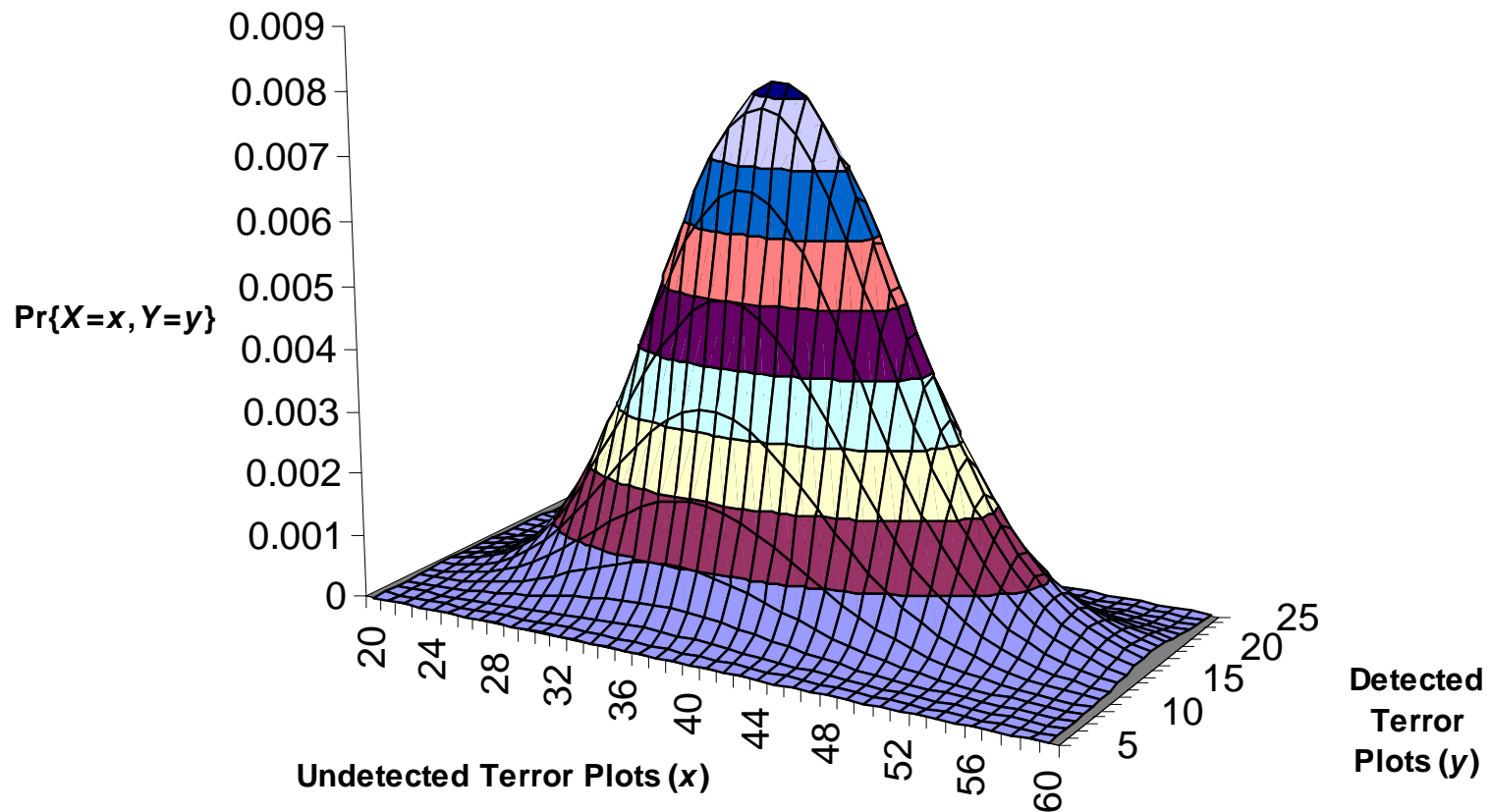
Generic balance equation:

$$(\alpha + \mu x + \rho y + \delta x(f-y))p_{xy} = \alpha p_{x-1,y} + \mu(x+1)p_{x+1,y} + \rho(y+1)p_{x,y+1} + \delta(x+1)(f-y+1)p_{x+1,y-1}$$

Also boundary equations for  $x=0$  and  $y=0, f$  plus probability conservation



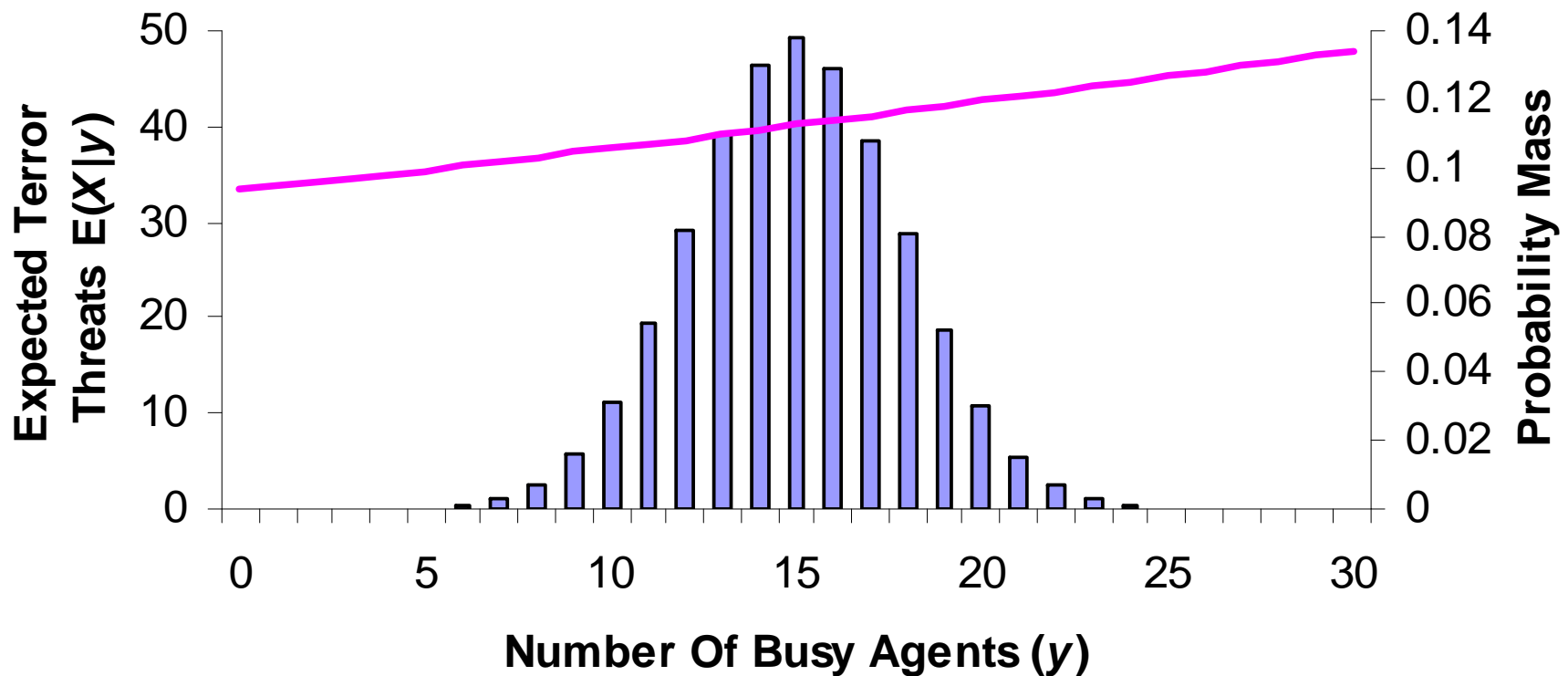
# Joint Distribution of Undetected ( $X$ ) and Detected ( $Y$ ) Terror Plots



Looks like bivariate normal distribution...

# Inference in Terror Queue Model

Inference In Terror Queue Model



Note that  $E(X|Y=y)$  is linear in  $y$

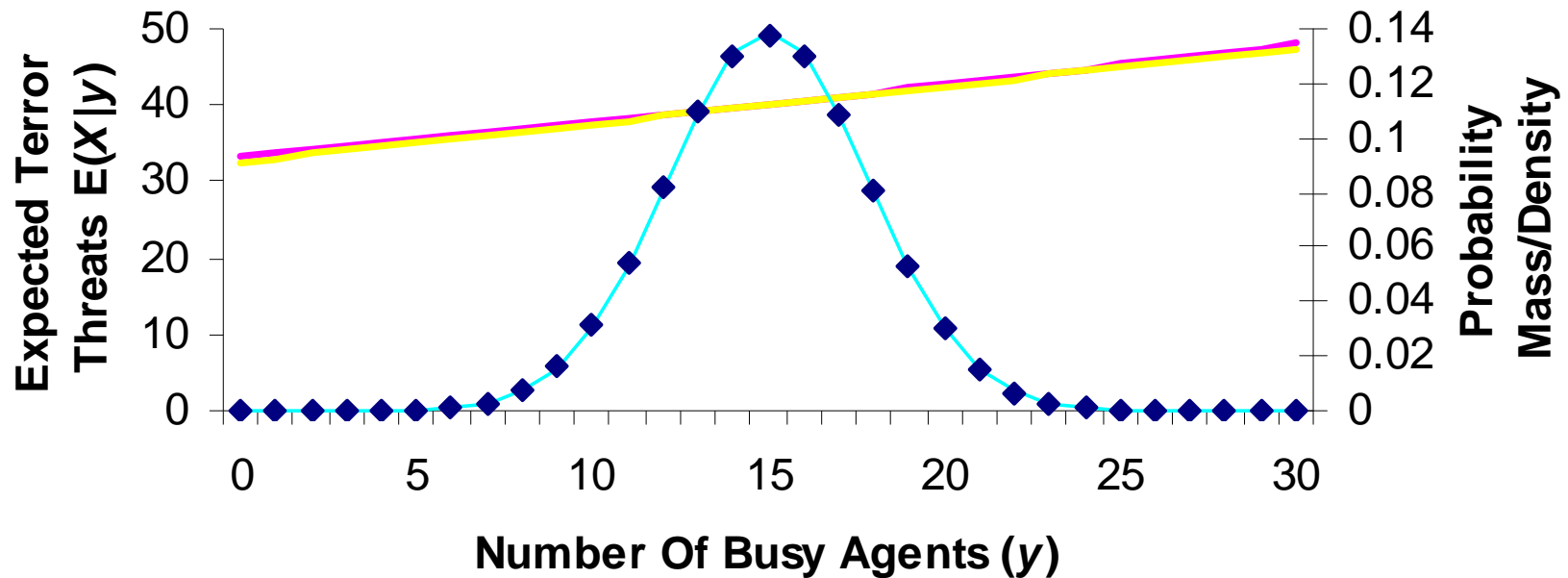
# Ornstein-Uhlenbeck Terror Queue

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- ◆ Motivated by approximate joint normality, formulate diffusion approximation (Barbour, *Adv Appl Prob* 8:296-314, 1976 among others)
- ◆ First formulate fluid model for expected number of undetected and detected terror threats
- ◆ Then construct diffusion approximation for joint stochastic fluctuations around expected values
- ◆ Instead of having to solve infinite system of linear equations as in Markov model, now only need to solve 2 nonlinear and 3 linear equations

# Comparing Markov and Diffusion Models for Hypothetical Example

## Inference In Terror Queue Model



—  $E(X|y)$  Markov —  $E(X|y)$  Diffusion ◆  $\Pr(Y=y)$  Markov — Diffusion

## In the US...

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- ◆ Since 9/11 the FBI “...increased the number of Special Agents working terrorism matters from 1,351 to 2,398.”

[http://www.fbi.gov/stats-services/publications/fbi\\_ct\\_911com\\_0404.pdf](http://www.fbi.gov/stats-services/publications/fbi_ct_911com_0404.pdf)

- ◆ Not all FBI Special Agents operate covertly, but other law enforcement agencies such as the New York Police Department also deploy undercover officers to disrupt terror plots
- ◆ Agents are “tip of the spear”

# Attack Level Staffing

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- ◆ How many agents  $f$  are needed to interdict a given fraction  $\theta$  of attacks?

# Attack Level Staffing

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$$f = \frac{\alpha}{\rho} \theta + \frac{\mu}{\delta} \frac{\theta}{1 - \theta}$$

- ◆ Can think of this as  $f = f_b + f_a$  where
  - $f_b = \alpha\theta/\rho$  is the number of busy agents ( $y$ ) and
  - $f_a = \mu\theta / (\delta(1-\theta))$  is the number of agents available for detection ( $f - y$ ), and solves

$$f_a \delta / (f_a \delta + \mu) = \theta$$

- ◆ For large  $\theta$ ,  $f_a \gg f_b$

# Other Staffing Objectives

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- ◆ Maximize the net benefits of preventing attacks, accounting for the cost of agents
- ◆ Allocate a fixed number of agents across different regions (or focusing on different terrorist groups) to prevent as many attacks as possible (or prevent as many attack casualties as possible)
- ◆ Game theory version – *terrorists select attack rate to achieve objectives*, recognizing optimal terror queue staffing



# Real Versus Fake Plots

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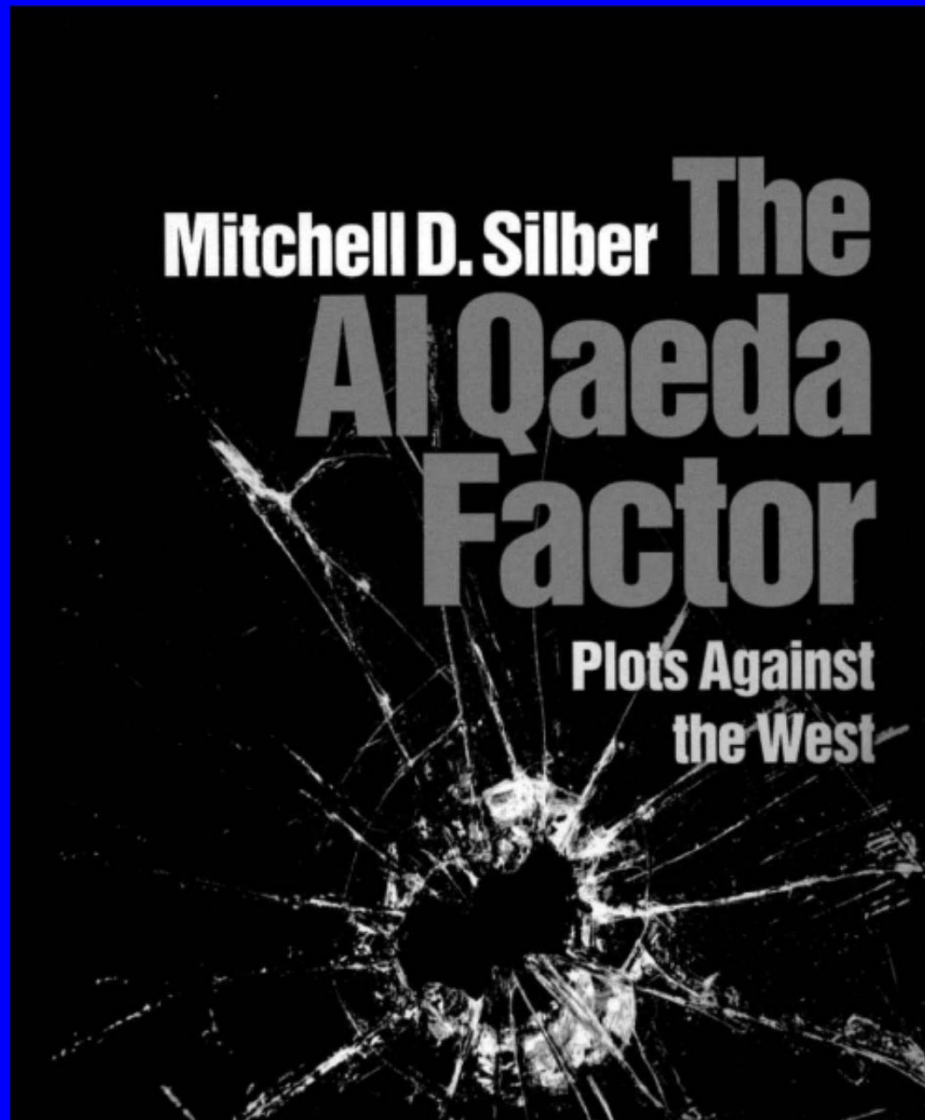
- ◆ Most information collected is unrelated to real terror threats:
  - “From July 2004 through November 2007, the FBI documented approximately 108,000 potential terrorism-related threats...The FBI determined that the overwhelming majority of the threat information...had no nexus to terrorism.” (FBI’s Terrorist Threat and Suspicious Incident Tracking System, US Dept of Justice, Audit Report 09-02, November 2008)

# Terror Queues with False Detection

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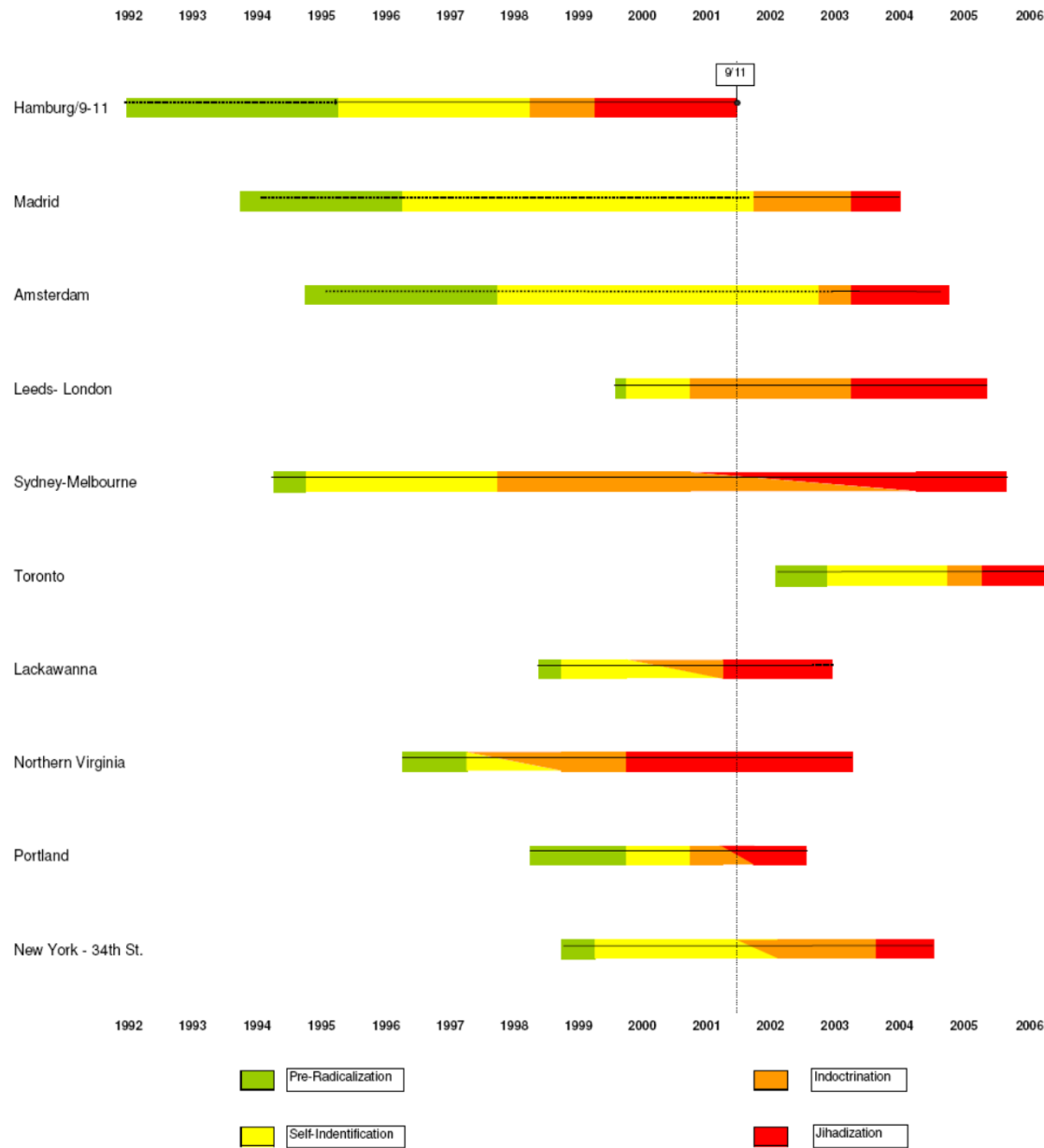
- ◆ Intelligence agents can make mistakes!
- ◆ In particular, agents can devote (perhaps considerable) effort to surveillance of persons not connected to terrorism
- ◆ Generating disinformation to divert agents from real to fake plots seems like a good strategy for terrorists to employ
- ◆ Can extend model to this case; reduces to prior results with smaller detection rate

# *Jihadi* Terror Plots in the United States



- ◆ “...most of the operations against the West have been manned by inspired volunteers who join it from the ‘bottom up’...”
- ◆ “...that al Qaeda Core’s role in plots is in general decline is a critical finding...”

### RADICALIZATION TIMELINE



Silber, MD and Bhatt, A  
(2007) *Radicalization in  
the West (NYPD)*

# Terror Plots in the United States

Center on Law and Security, New York University School of Law



## **Terrorist Trial Report Card:**

September 11, 2001-September 11, 2011

- ◆ The *Terrorist Trial Report Card* (TTRC) tracks and analyzes all federal criminal prosecutions since September 11, 2001 that the Justice Department claims are terror-related
- ◆ Data contain information regarding 1,054 prosecutions associated with 846 distinct defendants organized by date of indictment from 9/11/2001 through 6/30/2011

# *Jihadi* Terror Plots in the United States

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- ◆ 55% of TTRC cases are *Jihadi*
- ◆ TTRC's definition of *Jihadi* cases “...includes defendants who were formally or informally associated with an Islamist terror group -- whether one with a global jihadist ideology (i.e. Al Qaeda) or a local Islamist movement (i.e. Hamas). It also includes defendants unaffiliated with a terror group who aspired to such affiliation or who subscribed to a global jihadist ideology.”
- ◆ But, overwhelming majority of those prosecuted did not link to specific terror plots targeting the United States

# *Jihadi* Terror Plots in the United States

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- ◆ Review of *Jihadi* cases identified 26 cases linked to plans to attack Americans in US
  - thanks to NYPD's Mitch Silber for help eliminating non-plots, campfire plots, "let's play Jihadi" plots, etc.
- ◆ Cross check with Strom *et al* (2010) plus known attacks identified additional nine plots; 35 total
  - Sample includes: shoe bomber, captain underpants, Herald Square subway bomb, JFK fuel tanks, Time Square bomb, LAX shootings, etc.
  - Sample *excludes* Lackawanna 7, Bly Oregon camp, Northern Virginia Paintball, Atlanta casing plot, etc.

# Estimating the Duration of Terror Plots in the United States

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- ◆ When does a terror plot begin?
- ◆ Hard to know exactly; indeed terrorists probably don't know exact date either
- ◆ Futile to attempt pinpointing \*the\* start date
- ◆ Not futile to determine upper and lower bounds
  - “Early start” – plot had not begun before this date
  - “Late start” – plot had certainly begun as of this date
- ◆ Estimated early and late start dates from relevant court records such as indictments, criminal complaints, and other supporting legal documents in addition to media reports and other public sources



## E.g. Fort Dix Plot

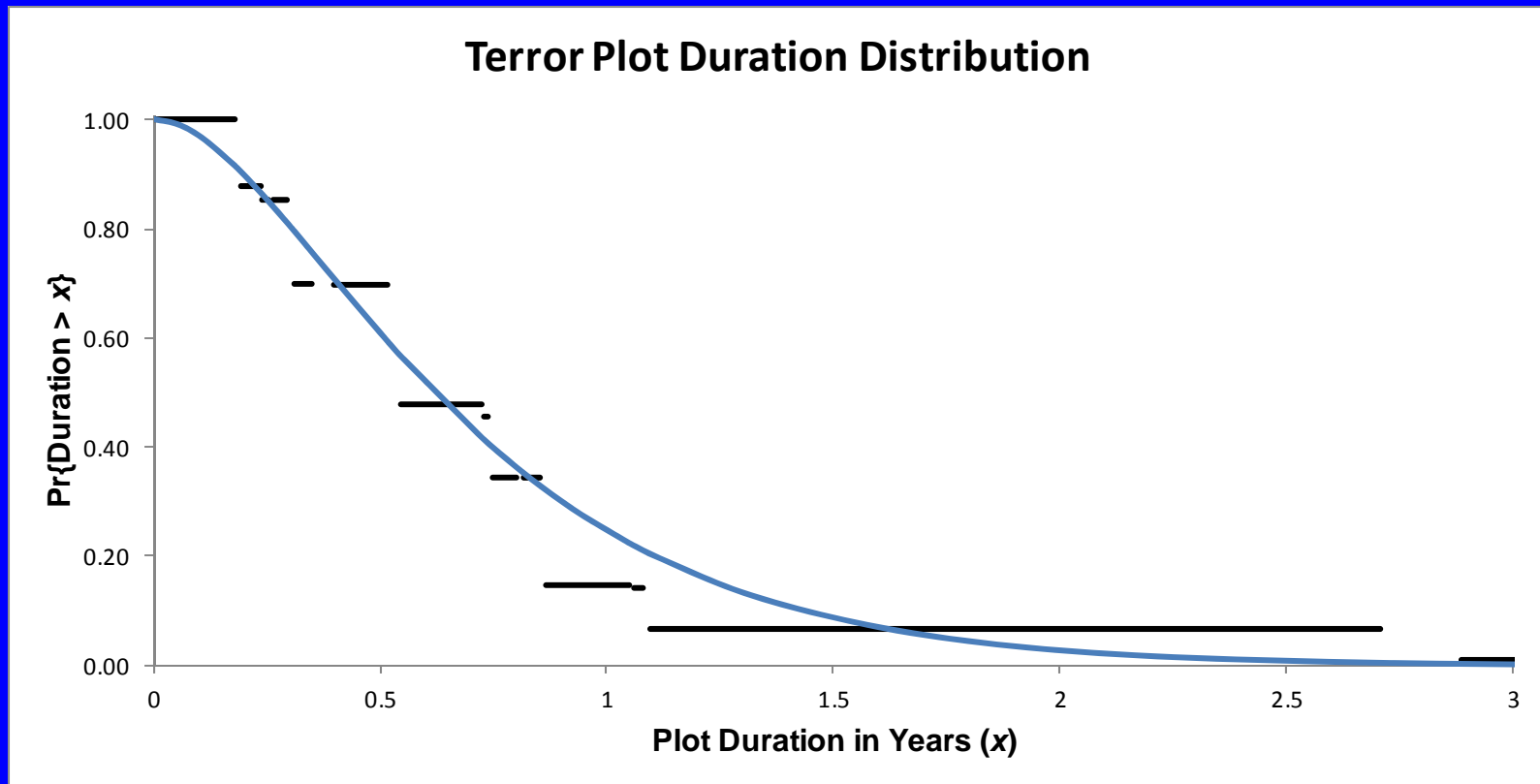
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- ◆ From criminal complaint, “On or about January 3, 2006, MOHAMAD SHNEWER, DRITAN DUKA, ELTVIR DUKA, SHAIN DUKA, and SERDAR TATAR conducted firearms training in Gouldsboro, Pennsylvania,”
- ◆ “On or about August 11, 2006, CW-1 (note: CW = cooperating witness) and MOHAMAD SHNEWER traveled to the Fort Dix military base to conduct surveillance...When CW-1 asked what made SHNEWER think of Fort Dix as a target, SHNEWER replied, ‘My intent is to hit a heavy concentration of soldiers...’ As SHNEWER and CW-1 drove into a specific area at Fort Dix, SHNEWER said, ‘...this is exactly what we are looking for. You hit 4, 5, or 6 humvees and light the whole place [up] and retreat completely without any losses.’ ”
- ◆ On this basis, early and late start dates were assigned to January 3 and August 11 respectively

**Table 1: Terror plot early start, late start and completion times**

<b>Plot</b>	<b>Key Convicts</b>	<b>Early Start (<math>\tau_e</math>)</b>	<b>Late Start (<math>\tau_l</math>)</b>	<b>End Date (<math>\tau_c</math>)</b>	<b>Source</b>
Shoe Bomb	Richard Reid	7/1/2000	7/7/2001	12/23/2001	13
Dirty Bomb	Jose Padilla	7/24/2000	7/1/2001	5/8/2002	14
Brooklyn Bridge	lyman Faris	1/1/2002	3/1/2002	3/19/2003	15
President Bush	Ahmed Omar Abu Ali	9/1/2002	9/30/2002	6/9/2003	16
Columbus OH shopping mall	Nuradin Abdi	1/1/2000	8/6/2002	11/28/2003	17
Shoulder fired missiles	Yasin Aref and Mohammed Mosharref Hossain	7/1/2003	11/20/2003	8/4/2004	18
Chicago courthouse	Gale William Nettles	7/1/2003	11/25/2003	8/5/2004	19
Herald Square subway	Shawaward Martin Siraj and James Elshafay	6/1/2004	6/30/2004	8/27/2004	20
Sell bomb to Al Qaeda	Ronald Allen Grecula	7/1/2002	12/31/2002	5/20/2005	21
California prison conspiracy	Kevin James and Levar Washington	12/1/2004	5/19/2005	8/31/2005	22
Chicago Sears Tower	Narseal Batiste <i>et al</i> ("Liberty City Seven")	11/1/2005	12/16/2005	6/22/2006	23
Liquid explosives on airlines	Abdulla Ahmed Ali <i>et al</i> ("Operation Overt")	12/15/2005	5/15/2006	8/9/2006	24
Rockford IL shopping center	Derrick Shareef	9/1/2006	11/29/2006	12/6/2006	25
Fort Dix NJ	Mohamed Ibrahim Shnewer <i>et al</i>	1/3/2006	8/11/2006	5/7/2007	26
Fuel tanks at JFK airport	Russell Defreitas <i>et al</i>	1/1/2006	8/1/2006	6/2/2007	27
South Florida pipe bombs	Ahmed Abdellatif Sherif Mohamed	1/1/2007	7/11/2007	8/4/2007	28
Bronx NY synagogue/Stinger missiles	James Cromitie <i>et al</i>	6/1/2008	4/10/2009	5/19/2009	29
Little Rock shootings	Abdulahakim Muhajahid Muhammad	11/14/2008	1/29/2009	6/1/2009	30
New York City subway	Najibullah Zazi	8/28/2008	12/1/2008	9/19/2009	31
Springfield IL courthouse	Michael C. Finton	11/11/2008	1/2/2009	9/23/2009	32
Fountain Place in Dallas TX	Hosam Maher Husein Smadi	1/1/2009	6/24/2009	9/24/2009	33
Fort Hood shootings	Nidal Hassan	12/1/2008	7/31/2009	11/5/2009	34
"Underwear bomb"	Umar Farouk Abdulmutallab	8/1/2009	10/15/2009	12/25/2009	35
Times Square car bomb	Faisal Shahzad	12/1/2009	2/25/2010	5/1/2010	36
Wrigley Field backpack bomb	Sami Samir Hasoun	5/29/2010	6/4/2010	9/19/2010	37
DC Metrorail	Farooque Ahmed	1/1/2010	5/15/2010	10/26/2010	38
Portland OR Christmas tree lighting	Mohamed Osman Mohamud	8/1/2009	8/19/2010	11/26/2010	39
Maryland military recruitment center	Antonio Benjamin Martinez	9/29/2010	10/22/2010	12/8/2010	40
Colorado dams and President Bush	Khalid Ali Aldawsari	3/11/2010	10/19/2010	2/23/2011	41
New York City synagogue	Ahmed Ferhani and Mohamed Mamdouh	10/15/2010	4/12/2011	5/11/2011	42

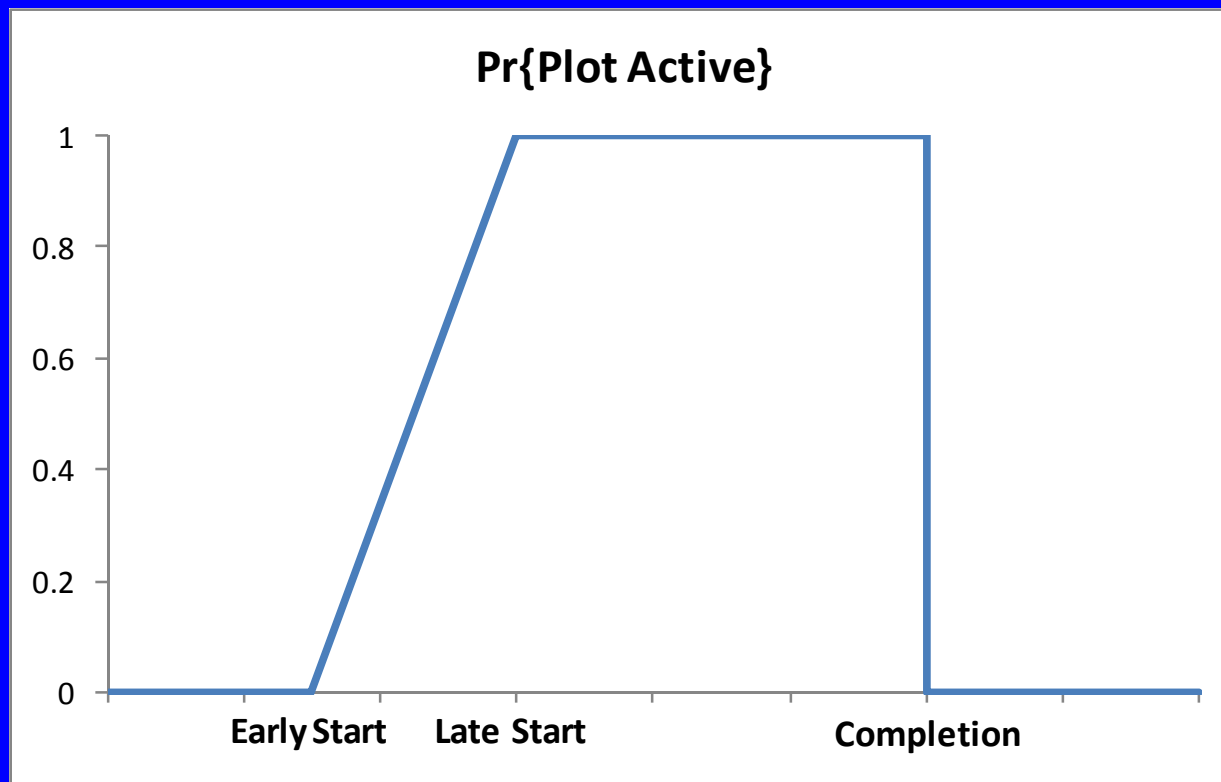
# Empirical US Jihadi Terror Plot Duration Distribution



- ◆ Erlang 2: mean=270 days (SE 43); nonparametric mean=268 days (SE 41)
- ◆ Erlang 2: 95% probability interval 33-750 days

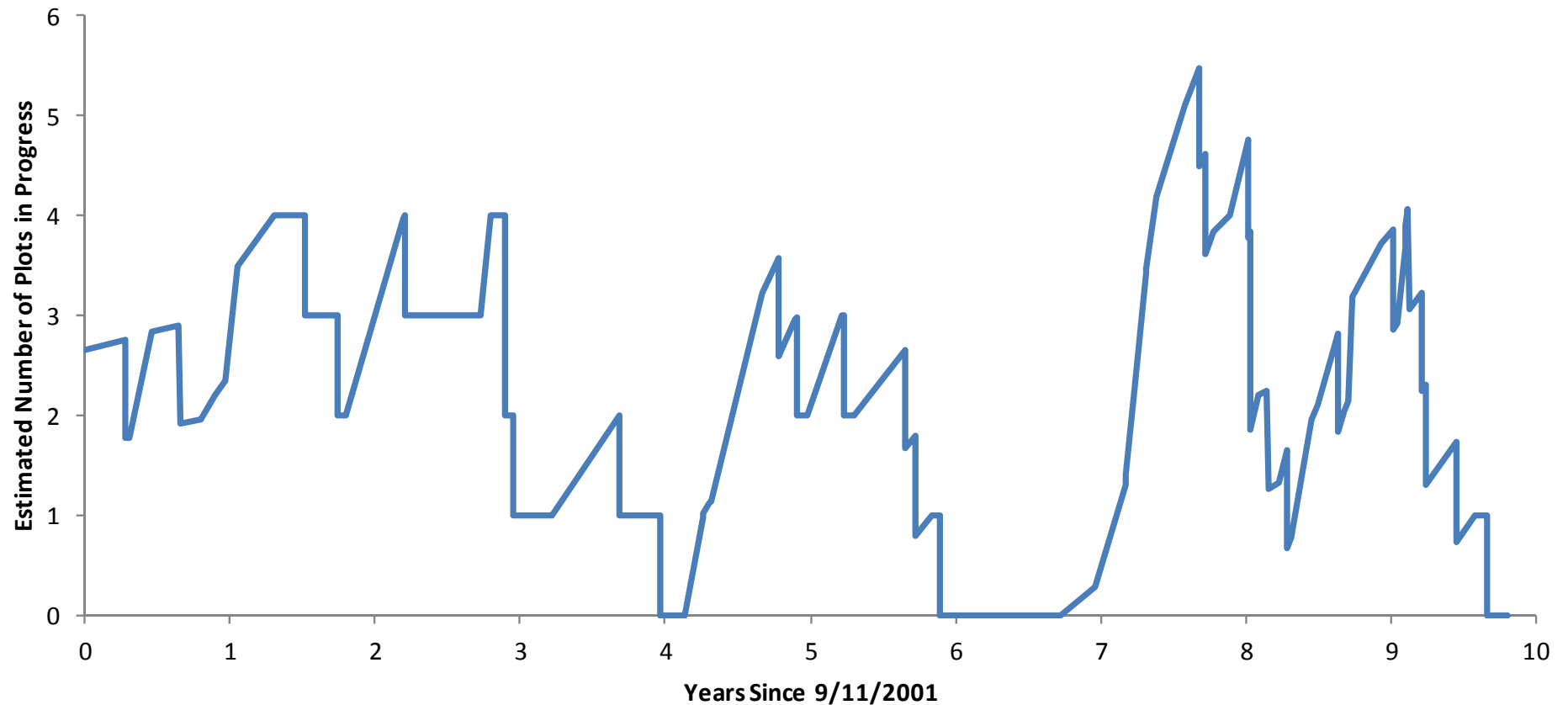
# Plots Over Time

- ◆ Let  $p(t)$  denote the probability that a particular plot is in progress at time  $t$
- ◆  $p(t)$  looks like...



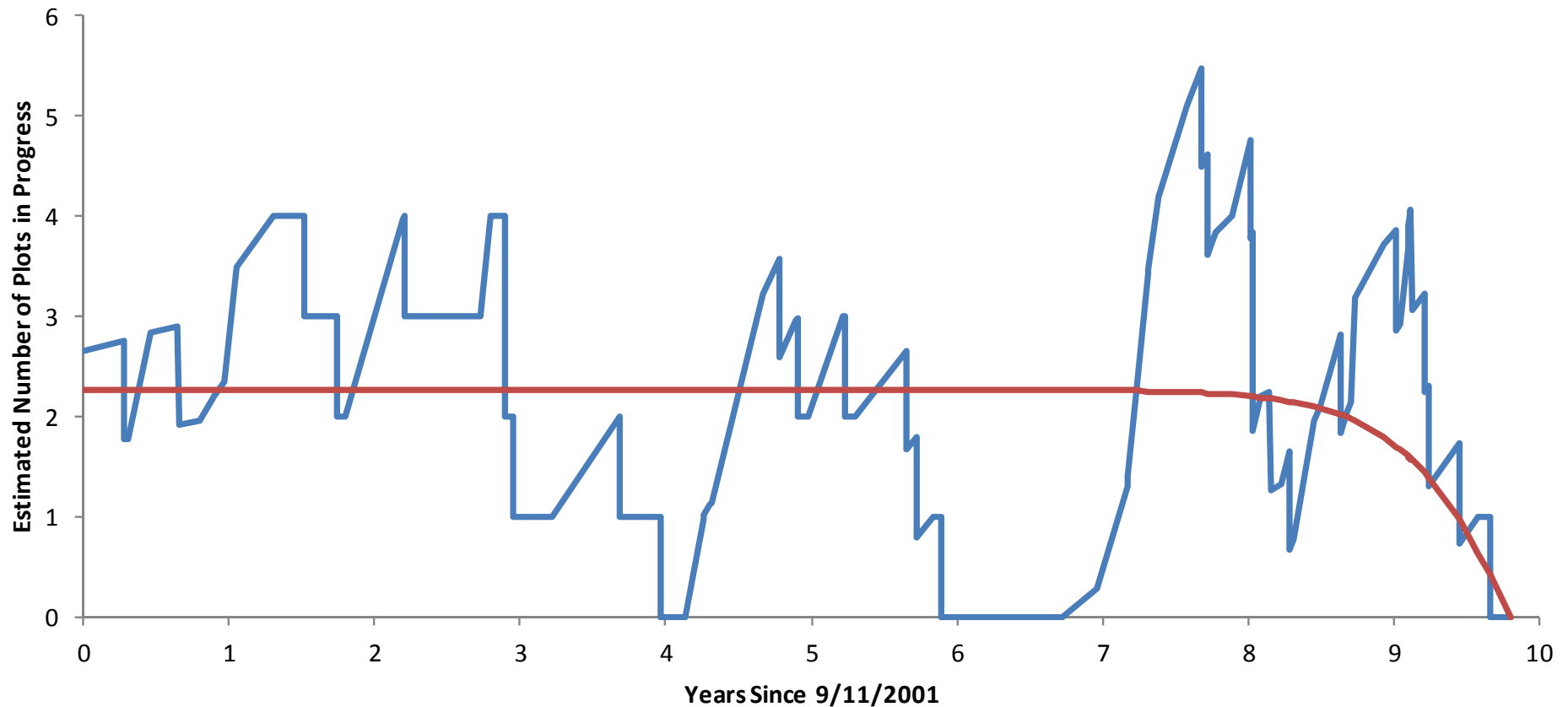
# Sum $p(t)$ Over All Plots...

## Estimated Number of Terror Plots in Progress



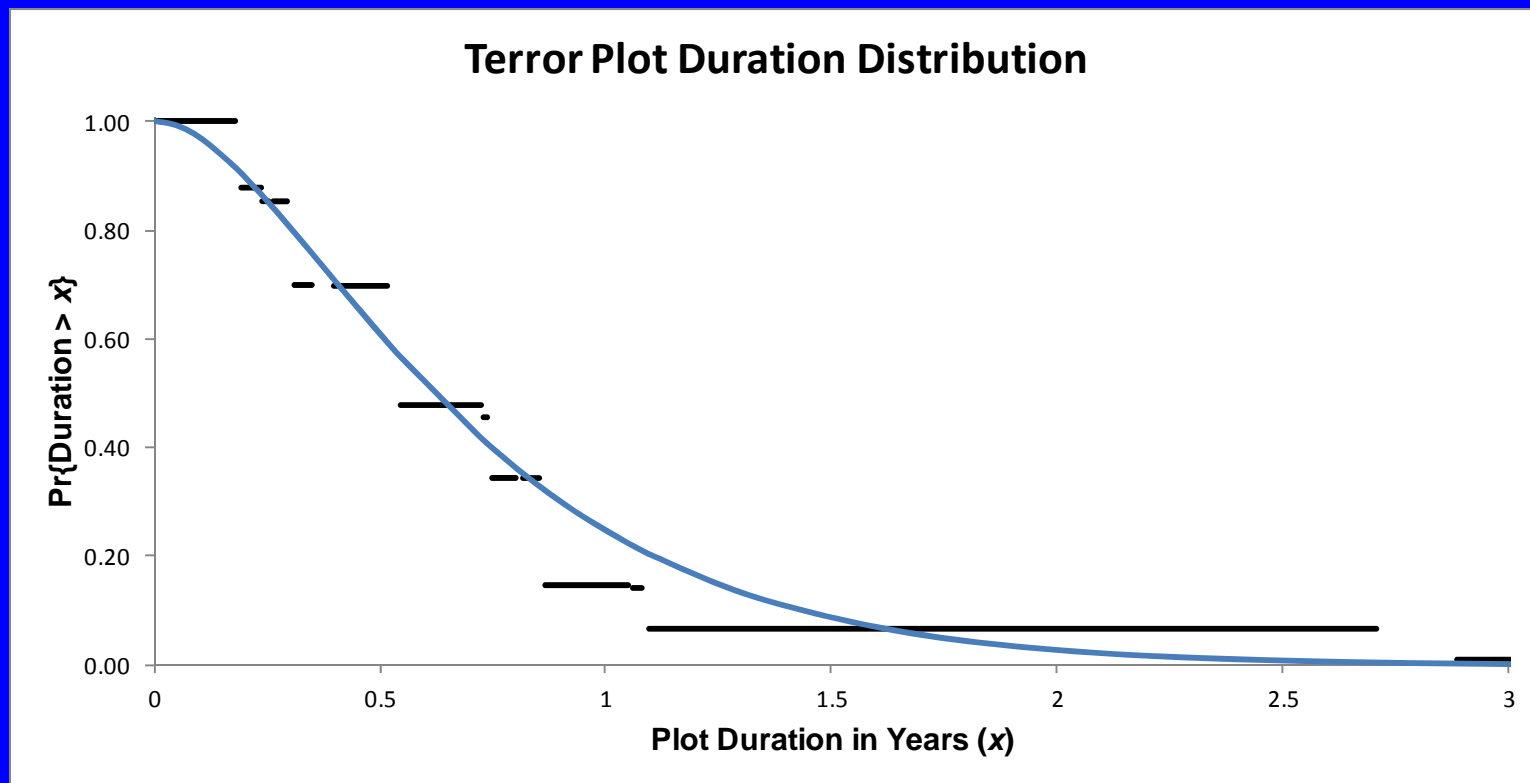
# Comparison To Terror Queue

Estimated Number of Terror Plots in Progress



# Terror Queue Staffing Presumes Exponential Plot Durations

- ◆ But just saw that in the US, probability distribution of time from plot arrival until attack or detection is not exponential



# Terror Queues With General Plot Duration Distributions

---

- ◆ Let  $T_D$  ( $T_A$ ) be the latent time from the start of a plot until detection (attack)
- ◆ Hazard functions for  $T_D$  ( $T_A$ ) given by  $f_a\delta(u)$  and  $\mu(u)$  respectively
- ◆ Let  $M = \min(T_D, T_A)$  be the plot duration with hazard function  $f_a\delta(u) + \mu(u)$

$$\Pr\{M > t\} = \exp\left(-\int_0^t (f_a\delta(u) + \mu(u))du\right)$$

$$\Pr\{\text{Detect}\} = \int_0^\infty f_a\delta(u) \Pr\{M > u\}du$$

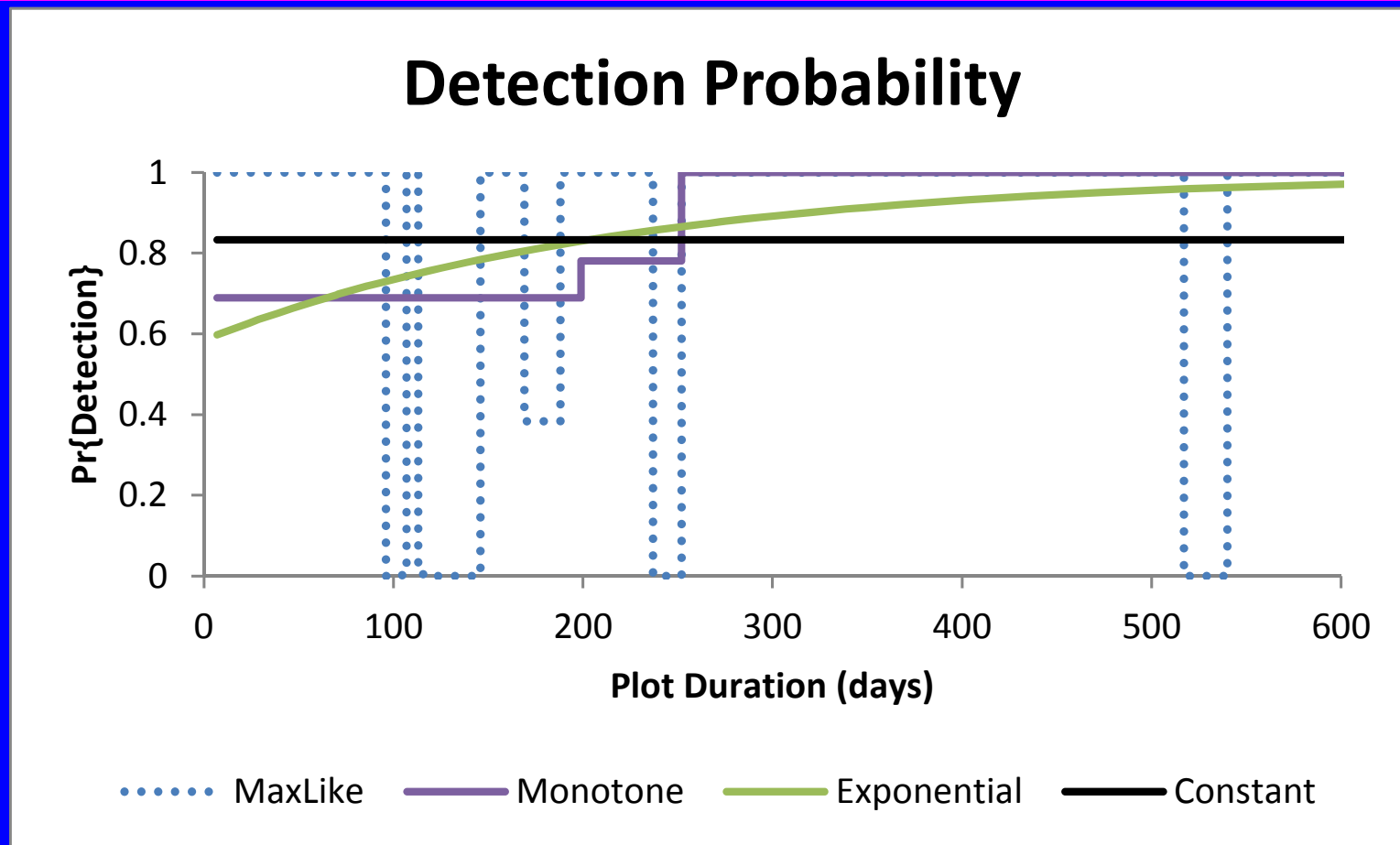


# Looks Rough For Staffing Model

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- ◆ Absent special structure (e.g. constant hazards as in original terror queue model), need knowledge of plot-age-dependent hazards for attack and detection for attack-level staffing
- ◆ Might there be an intermediate, defensible assumption between constant and arbitrary terror plot duration hazards that leads to simple staffing models?

# Duration Dependent Detection?



- ◆ Likelihood ratio tests: cannot reject hypothesis that  $p_D(u) = p_D$  (i.e. constant!)

# Plot Durations With Proportional Hazards

---

- ◆ When is a plot more likely to be detected?
- ◆ When there is more plot activity
- ◆ A good measure of plot activity is attack hazard!
- ◆ So, take the attack hazard as “baseline,” and take detection hazard as proportional to baseline
- ◆ That is, assume  $\delta(u)$  is proportional to  $\mu(u)$
- ◆ This yields constant detection probability with age of plot, and hence constant detection probability overall

# Plot Durations With Proportional Hazards

---

- ◆ Recall that  $M = \min(T_D, T_A)$  is the plot duration with hazard function  $f_a\delta(u) + \mu(u)$
- ◆ Assuming that  $\delta(u)$  is proportional to  $\mu(u)$  implies that

$$\Pr\{\text{Detection}\} = p = f_a\delta(u) / (f_a\delta(u) + \mu(u))$$

is constant with the age of the plot

# Staffing With Proportional Hazards

---

- ◆ The proportional hazards assumption is

$$\delta(u) = k \mu(u)$$

- ◆ This implies that

$$\begin{aligned} p &= \frac{f_a k \mu(u)}{f_a k \mu(u) + \mu(u)} \\ &= \frac{f_a k}{f_a k + 1} \end{aligned}$$

# What About Busy Agents?

---

- ◆ Recall that  $f_b$  is the expected number of busy agents
- ◆ If on average it takes  $1/\rho$  time units to interdict detected plots, a fraction  $p$  are detected, and the attack rate equals  $\alpha$ , then as before we have

$$f_b = \alpha p / \rho$$

# Attack Level Staffing Formula

---

- ◆ Recall the decomposition  $f = f_b + f_a$
- ◆ For attack level staffing, set

$$f_b = \alpha\theta/\rho$$

- ◆  $f_a$  solves  $k f_a / (k f_a + 1) = \theta$ , that is

$$f_a = \frac{1}{k} \frac{\theta}{1-\theta}$$

- ◆ Overall attack level staffing then equals

$$f(\theta) = \frac{\alpha\theta}{\rho} + \frac{1}{k} \frac{\theta}{1-\theta}$$

## Special $k$ ?

---

- ◆ If you know fraction of plots detected for *some* staffing level  $f^*$ ,  $p(f^*)$ , can set

$$f_a^* = f^* - \alpha p(f^*)/\rho$$

and set  $k$  equal to

$$k = \frac{1}{f_a^*} \frac{p(f^*)}{1-p(f^*)}$$

- ◆ Expect  $\alpha p(f^*)/\rho$  to be small; can often ignore



# Example

---

- ◆ Recall that in US have detected 80% of Jihadi terror plots
- ◆ FBI reported have assigned  $\approx 2400$  special agents to terrorism
- ◆ Take  $f_a^* = 1600$  for this example
- ◆ Special  $k$  given by  $(1/1600) * .8 / .2 = 1/400$
- ◆ If doubled available agents to 3,200 would prevent  $(3200/400) / (3200/400 + 1) = 8/9 \approx 89\%$

# Example

---

- ◆ Want to prevent 95% of Jihadi plots
- ◆ Using staffing formula given prevent 80% with  $f^*=1,600$ , would need

$$f_a = \frac{1}{1/400} \times \frac{.95}{1-.95} = 7,600$$

- ◆ Is it worth it?

$$\max_{0 \leq \theta < 1} b\alpha\theta - cf(\theta)$$

# What If Don't Know $f^*$ ?

---

- ◆ Suppose all you know is current probability of detection  $p$
- ◆ Want to increase this by  $100\varepsilon\%$
- ◆ Using staffing formula, easy to show that need to increase number of agents by

$$100 \frac{\varepsilon}{1 - (1 + \varepsilon)p} \%$$

# Example

---

- ◆ Don't really know how many agents there are, but know now catching 80% of plots
- ◆ Suppose want to catch 95%, an increase of 18.75% in the detection probability
- ◆ Need to increase existing covert force by

$$100 \times \frac{0.1875}{1 - (1 + 0.1875) \times 0.8} = 375\%$$

(that is, a factor of 4.75)

# Example: Allocate Agents Across Groups

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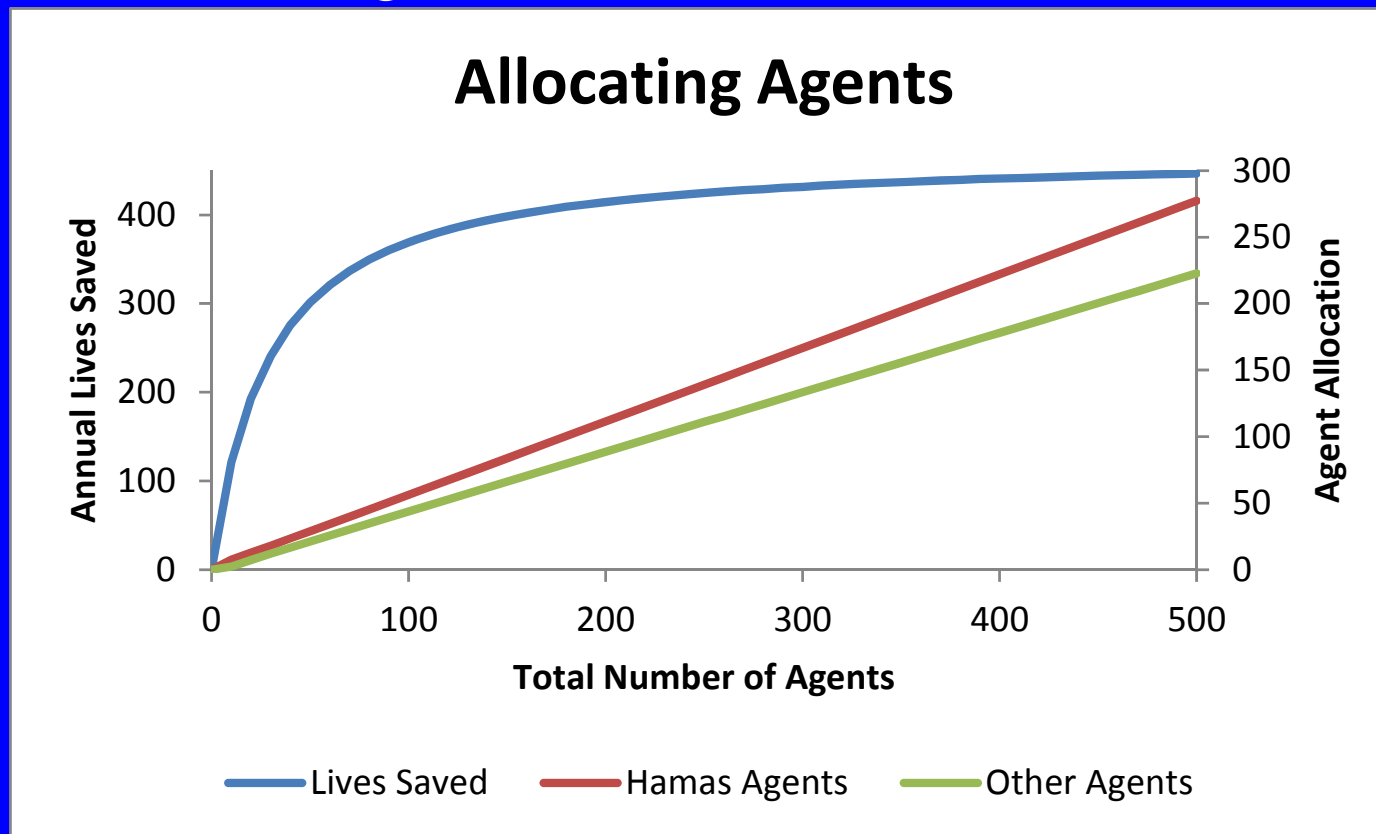
- ◆ Suppose have  $n$  different geographic regions/groups
- ◆ Constrained to  $f$  agents in total
- ◆ How to allocate agents across groups?

$$\begin{aligned} \max \quad & \sum_{i=1}^n b_i \alpha_i \theta_i \\ \text{st} \quad & \sum_{i=1}^n f_i(\theta_i) \leq f \end{aligned}$$

$$0 \leq \theta_i < 1 \text{ for } i = 1, 2, \dots, n.$$

# Intifada Example

- ◆ Hamas suicide bombers killed 8.9 civilians/attack (other groups 3.5)
- ◆ Allocate agents to maximize lives saved



# Summary

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- ◆ Terror queue framework connects attempted attacks to outcomes via detection/interdiction by undercover agents
- ◆ Analysis of available data suggests duration distribution for *Jihadi* plots in the US
- ◆ Same data suggest that hazard functions for time to detection/attack are proportional
- ◆ Sensible if detection more likely when terrorists more active, and attack hazard marks terrorist activity

# Summary

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- ◆ Proportional hazards assumption enables simple staffing models that do not otherwise depend on the specific probability distributions of times to detection or attack
  - Attack level staffing; force allocation; even game theoretic version where terrorists strategically select attack rates
- ◆ Models do assume agent times to detection are mutually independent
  - Correlation across times to detection equivalent to reducing number of independent agents
- ◆ Models exhibit strong diminishing returns in attack detection as # agents increases